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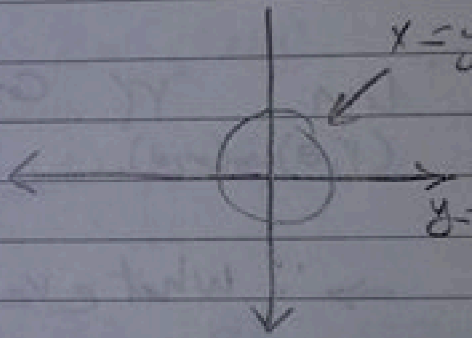
{ limits }

* $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ $\Rightarrow$ for the path $x=y$

$$\therefore \lim_{y\to 0} \frac{y^2}{2y^2} = \frac{1}{2}$$



$\Rightarrow$ for the Path $y=0$

$\therefore \lim\limits_{x\to 0} \dfrac{0}{x^2+0} = \dfrac{0}{0}$ غير موجود

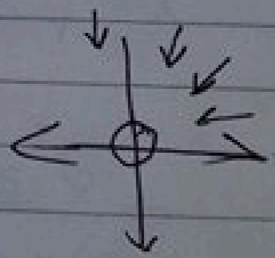
$\therefore \Rightarrow$ the limit is not Exisit.

another Solution

put $x = r\cos\theta$ $\qquad y = r\sin\theta$

$$\therefore \lim_{(r,\theta)\to(0,0)} \frac{r^2\cos\theta\, r\sin\theta}{r^2} = \frac{1}{2}\lim_{(r,\theta)\to(0,0)} 2\cos\theta\sin\theta$$

$$\therefore \frac{1}{2}\lim_{\theta\to 0} \sin 2\theta$$



Put $\theta = 0, 20, 90 \ldots\ldots$

$\therefore$ the limit is not exisit

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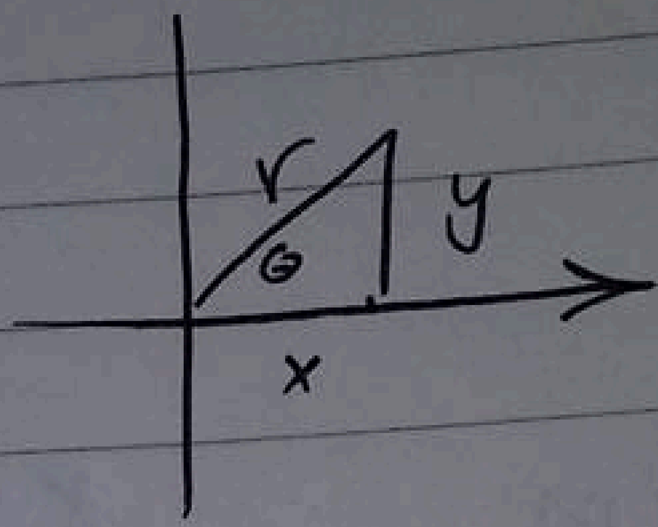


(2) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+y^3}{x^2+y^2}$

هنحول الصورة الكارتيزية لل r,θ

$x = r\cos\theta$

$y = r\sin\theta$

$$\lim_{(r,\theta)\to(0,0)} \frac{r^3\cos^3\theta + r^3\sin^3\theta}{r^2}$$

$$\lim_{(r,\theta)\to(0,0)} r(\cos^2\theta + \sin^2\theta) = 0$$

$\Rightarrow \because$ whatever the value of θ $\lim \to 0$

* Partial of Differentiation التفاضل الجزئي

$* \; z(x,y) = x^2y + 2yx$

$\dfrac{\partial z}{\partial x} = 2xy + 2y$ | $\dfrac{\partial^2 z}{\partial x^2} = 2x$

$\dfrac{\partial z}{\partial y} = x^2 + 2x$ | $\dfrac{\partial^2 z}{\partial y^2} = \text{zero}$

$\Rightarrow \dfrac{\partial^2 z}{\partial x \partial y} = 2x + 2$

هنفاضل z بالنسبة لـ x
والناتج نفاضله بالنسبة لـ y
أو العكس ... الناتج النهائي هيطلع واحد

$$\frac{\partial z}{\partial x} = z_x = \partial_x z$$

$$\frac{\partial z}{\partial y} = z_y = \partial_y z$$

$$* \; \frac{\partial^2 z}{\partial x^2} = z_{xx} = \partial_x^2 z$$

$$* \; \frac{\partial^2 z}{\partial x \partial y} = z_{xy} = \partial_{x,y}^2 z$$

* Chain Rule

$$\begin{tikzcd} & z \arrow[dl] \arrow[dr] & \\ x \arrow[d] & & y \arrow[d] \\ t & & t \end{tikzcd}$$

$$z = f_n(x, y)$$

$$y = f_n(t) \;,\; x = f_n(t)$$

$$\therefore \Rightarrow \frac{dz}{dT} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial T} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial T} = \underline{\nabla} z \odot \underline{v}$$

ممكن نكتبها $\frac{dy}{dT}$ علشان y دالة في متغير واحد هو t

Directional derivative (Nabla) ∇ — المشتقة الاتجاهية.

$$\boxed{\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{dy}, \frac{\partial}{\partial z}\right)}$$

$$\Rightarrow z = xy^2 \quad (\text{Scalar})$$

$$\nabla z = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial z}{\partial z}\right)$$

$$\nabla z = (y^2, 2xy, 0) \quad (\text{Vector})$$

الـ Nabla بتدخل ع الدالة بتحولها من
دالة قياسية scaler
لـ دالة متجهة vector.

$$\nabla z = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right)$$

$$\underline{v} = \left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}\right)$$

$$\therefore \frac{dz}{dt} = \underline{\nabla} z \odot \underline{v}$$

الـ Nabla لو دخلت على دالة قياسية بتحولها لمتجه

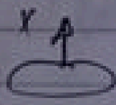
ولو دخلت على دالة Vector بتحولها لـ tensor

$$\text{tensor} = A = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$$

$A_{xx} \rightarrow$

σ_{xx} ⇐ الاجهاد الي بيأثر على المستوى العمودي على محور x في اتجاه x

(اجهاد شد او ضغط)



σ_{xy} ⇐ " " " " " " " x في اتجاه y

" اجهاد قص

المعنى الهندسي لـ ∇

∇f هو متجه يعبر هندسياً عن العمودي على السطح

ex:- $f(x,y,z) = x^2 + z^2 - 9 + y^2$

أو $x^2 + y^2 + z^2 = 9$

⇒ Find the unit normal to the geometry $f(x,y,z)$ at the poit $(1, 1, \sqrt{7})$

Gradiant الانحدار: النقطة $(1,1,\sqrt{7})$ تقع على معادلة الكرة

⇒ $\nabla f(x,y,z) = (2x, 2y, 2z)$ ⇐ دالة العمودي ع سطح الكرة.

$\nabla f\big|_{(1,1,\sqrt{7})} = (2, 2, 2\sqrt{7})$ ⇐ المتجه العمودي على سطح الكرة عند نقطة $(1,1,\sqrt{7})$.

$$\Rightarrow |\nabla F| = \sqrt{4+4+28}$$

unit $\leftarrow$ $$\hat{\underline{\nabla F}} = \frac{\nabla F}{|\underline{\nabla f}|}$$

ex:- If $z(x,y) = xy^2$, $x = fn(t)$
$y = fn(t)$

find the total Change in z when $(x,y) \longrightarrow (1,2)$ and the velocity of (x,y) was $(3,4)$

عايز معدل تغير z
لما يتغير سرعة النقطة
$(3,4) = (x,y$

Solution

$$\nabla z = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = (y^2, 2xy)$$

$$\underline{v} = (3,4)$$

$$\frac{dz}{dt} = \underline{\nabla z} \cdot \underline{v} = (y^2, 2xy) \cdot (3,4)$$

$$\frac{dz}{dt} = 3y^2 + 8xy$$

at $(1,2)$ $$\therefore \frac{dz}{dt} = 12 + 16 = 28$$